CPS331 Lecture: Fuzzy Logic last revised October 3, 2018

Objectives:

1. To introduce fuzzy logic as a way of handling imprecise information

Materials:

1. Projectable of "young" membership function
2. Projectable of crisp representation of the height of men (Negnevitsky Figure 4.3)
3. Projectable of fuzzy representation of the height of men (Negnevitsky Figure 4.3)
4. Projectable of Figure 3.2 from Schmucker

I. The Issue of Imprecise Information

A. We have noted that knowledge representation is a key problem in AI. Several issues pose a particular challenge for symbolic knowledge representation schemes. One of these arises from the fact that in formal logic (the basis of many representation schemes), statements are either true or false. But that is not often the way things are in everyday life - especially when representing information expressed in natural language.

1. Consider the word "tall" in English.

   a) Most of us would say that a man who is 6'5" is definitely tall, and a man who is 5'0" is definitely not.

   b) But what about a man whose height is 6'0". Is this person tall or not? The problem is that there is a sense in which he is tall and a sense in which he isn't tall.

2. Many concepts - like tallness - are inexact or "fuzzy". It is not the case that all people are either tall or not tall. Some are "sort of tall"

   What are some other examples of inherently inexact (fuzzy) concepts?

   ASK

B. Notice that the problem of inexact knowledge is distinct from the problem of uncertain knowledge, which we looked at earlier in the course.

1. We may know with absolute certainty that a person's height is 6'0" - but it's still unclear whether we should call him tall.

   That is, a concept may involve factual certainty but still be inexact.

2. Conversely, it is also possible for a given piece of information to be exact but uncertain. For example, a witness to a crime might report being "pretty sure" that the criminal was a male.

   If the criminal's gender were known with certainty, it would be either male or female; this concept is uncertain but not inexact.

3. Of course, knowledge can be both uncertain and inexact. We may be reasonably sure that a certain person is in his 40's, but be unable to pinpoint the exact age and also be uncertain that his actual age is not something else (i.e. he may be in his 20's but have led a debauched life that makes him look older!)

   Is this person old? In this case, we are dealing with a concept that is both uncertain and exact.

C. There are many problems where we need to be able to work with imprecise information. While probability theory can sometimes be used in such cases, often the rigor of strict probabilities is not reasonable.

   Example: If a man were 6'0" tall, how would you assign a meaningful probability to the statement "this man is tall"?

II.Fuzzy Set Theory

A. The problem of inexact knowledge has given rise to various systems of multivalued logic, in which a proposition can have values other than just true and false. We will look at one such logic, - called fuzzy logic (which is not what a professor might write in red ink on a paper :-))

   The most fruitful work in the area of multivalued logics has come to be known as <u>fuzzy set theory</u>, and these inexact concepts are therefore sometimes called fuzzy concepts. This term is used in contrast to conventional logic which is called <u>crisp logic</u> by way of contrast.

B. Note that there is a very close connection between mathematical sets and logic predicates of one argument- which are equivalent to functions of one argument whose value is either 0 or 1 (false or true).

   1. Given any predicate of one argument, we can construct a set whose members are the objects that satisfy that predicate.

      Example: The attends_gordon predicate defines a set - the set of all students at Gordon.

   2. Conversely, given any set we can define a membership predicate, which is true for a given object just when that object is a member of the set - or a membership function (called the characteristic function for the set) whose value is either 0 or 1.

      Example: Given the set of students enrolled in CPS331, we could define the enrolled_in_cps331 predicate that is true of them but no one else. or a function whose value is either 0 or 1.

      (Show how this works out for a student that is in the class and for one that is not)

   3. Fuzzy set theory extends this concept to a <u>membership function</u> whose value lies between 0 and 1.

a) A crisp set is defined by a membership function whose value is always either 0 or 1 - 1 if its argument is a member of the set, and 0 if it is not.

b) A fuzzy set is one whose membership function can have any value between 0 and 1. In particular, it is 0 for any object that is certainly not in the set, 1 for any object that certainly is in the set, and some value in between for other objects that are "sort of" in the set. The value of the function, then, is a measure of the degree of membership in the set.

(1)Example: Consider the following set:

A = { x | x is a natural number and Mary's car can hold x adult passengers in addition to the driver }

Suppose, further, that Mary's car is a subcompact. Clearly, then:

1, 2, and 3 are elements of A, and 7, 8, and 9 are not.

But what about 4, 5, and 6? Certainly, 4 is more in the set than 5 is, and 5 more than 6. Perhaps, then, we might agree to associate a degree of membership of 0.8 with 4, 0.5 with 5, and 0.2 with 6. We could write the set A as follows:

A = { 1.0 / 1, 1.0 / 2, 1.0 / 3, 0.8 / 4 , 0.5 / 5, 0.2 / 6 }

(Note that we don't list elements whose degree of membership is zero - otherwise, we'd have to list a lot of numbers!)

(Note: some writers write a fuzzy set element in the opposite order from what we have used here: value / degree of membership)

(2)Example: Suppose we want to define the following set:

A = { x | a person x years old is a young person }

We might define the membership function for this set as follows:

m = 1.0 for x <= 25

m = 1 / (1 + ((x-25)/5)^2) for x > 25

- e.g. for someone 30, m = 1 / (1 + 1) = 0.5

for someone 40, m = 1 / (1 + 9) = 0.1

etc.

(Both above examples based on Schmucker)

This can be pictured graphically, as follows:

PROJECT

c) Note that a fuzzy membership function must **<u>NOT</u>** be confused with a probability, even though both take values in the range 0 .. 1

(1)Probability is defined rigorously in terms of the fraction of a population that has some characteristic.

(2)Fuzzy membership functions are more add-hoc, and have no relationship to distributions of a characteristic in a population.

Example: we may choose to regard a man having a height of 5'10" as being in the set of tall people with degree of membership of, say, 0.5. This is not saying anything about the distribution of heights in the population!

(3)One way to define a membership function, though, is in terms of what fraction would say a given property holds for a given value - e.g. if 50% of people would say that a man who is 6'0" tall is tall, then we would include the value 0.5 / 6'0" in the set representing the heights of tall men. Of course, this is not the same thing as saying that the probability that a 6'0" person is tall is 0.5!

C. As in ordinary set theory, we want to be able to define the operations of set intersection and union on fuzzy sets, and we want to be able to define complementation - the set of all items NOT in some fuzzy set.

1. It can be shown that the only natural definition of fuzzy set intersection (i.e. the only one that preserves properties like the distributive property is this:

   Let $S = A \cap B$,

   where S has membership function m_S, A has membership function m_A, and B has membership function m_B.

   Then $m_S(x) = \min(m_A(x), m_B(x))$.

   a) Example: Let A be a fuzzy set of athletic people, and T be a fuzzy set of tall people

      Develop A and T for people in the class. Be sure that at least one person is totally excluded from A and at least one from T. Be sure at least one person is totally included in each.

   b) Now let $S = A \cap T$ be the fuzzy set of athletic, tall people. Develop.

   c) Note that, as appropriate, $m_S(x)$ is 0.0 if x is totally excluded from either A or T, which means x is excluded from S as well.

2. Similarly, it has been shown that the only natural definition of fuzzy set union is this:

   Let $S = A \cup B$,

   where S has membership function m_S, A has membership function m_A, and B has membership function m_B.

   Then $m_S(x) = \max(m_A(x), m_B(x))$.

a) Example: Continue the above - let S be the set of people who are either athletic or tall - i.e.$S = A \cup T$. Develop for the class examples.

b) Note that, as appropriate, $m_S(x)$ is 1.0 if x is totally included in either A or T.

3. There is no provably natural definition for the complement, but the following is generally used:

   Let $S = \neg A$ (the set of all things that are not members of A), where S has membership function m_S and A has membership function m_A

   Then $m_S(x) = 1 - m_A(x)$

   a) As appropriate, if some object x is totally a member of A ($m_S(x) = 1.0$) then it is totally excluded from A' ($m_S(x) = 0.0$); and if it is totally excluded from A then it is totally included in A'. The latter depends on having some notion of the "universe" - i.e. the set of all elements that might be in some set.

   b) Example: Develop $\neg A$ and $\neg T$ for the class example, using the entire class as the universe.

   c) However, consider the intersection $A \cap A'$. In ordinary logic, this should be the empty set - but in fuzzy logic, an element x is a member of $A \cap A'$ with degree of membership $\min(m_A(x), (1-m_A(x))$. This assumes its maximum value when $m_A(x) = 0.5$, allowing some elements to be both in A and A' with degree of membership as large as 0.5!

      That is, in fuzzy logic the law of non-contradiction does not hold.

   d) Again, consider the union $A \cup A'$. In ordinary logic, this should be the universe - but in fuzzy logic, an element x is a member of

$A \cup A'$ with degree of membership
$\max(m_A(x), (1-m_A(x))$. This assumes its minimum value when $m_A(x) = 0.5$, allowing some elements to be in both in A and A' with degree of membership as small as 0.5!

That is, in fuzzy logic the law of the excluded middle does not hold either.

D. Since set intersection corresponds to logical AND, set union to logical OR, and set complementation to logical negation, we can use the rules for fuzzy intersection, union, and complementation to combine degrees of membership in systems that deal with fuzzy concepts.

Example: Develop evaluation of `athletic(X) ^ tall(X)` for various people in the class.

III. Modeling Linguistic Variables as Fuzzy Sets

A. For the sake of user-friendliness, it is desirable for a system to dialog with humans (both input and output) using natural language terms that are inherently imprecise.

1. This might also be the case in the formulation of rules for an expert system - we might want to allow an expert to use terms like "high", "medium" and "low" instead of numeric quantities.

2. Natural language terms fall into two main categories:

    a) Primary terms that express basic concepts (e.g. terms like "low", "medium", or "high" or "cold", "warm", "hot" or the like)

    b) Linguistic hedges that modify primary terms (e.g. "very", "sort of", etc.)

B. We can model primary terms by using fuzzy sets.

Example - a standard example in describing fuzzy set theory is to use the concepts "short", "medium", and tall applied to the heights of men.

1. If we were using traditional crisp logic, we would have to use specific thresholds - e.g. we might decide that any man with a height less than 170 cm (about 5'8") is short; between 170 and 180 cm (about 6'0") is average, and over 180 cm is tall. The resultant membership function would look like this:

   PROJECT Crisp representation of short, average, and tall men

2. If we used a fuzzy representation, we might use something like this:

   PROJECT Fuzzy representation for short, average, and tall men

3. Note that a consequence of using the fuzzy representation is that the sets overlap. So, for example, a man whose height is 184 cm (a bit more than 6'1") would be a member both of the set of average height men (with membership 0.1) and of the set of tall men (with membership 0.4)

C. Another Example - if our domain of discourse were the natural numbers between 1 and 10, we might model various quantity words by fuzzy sets like the following (examples from Schmucker 1982)

1. few = { .4/1, .8/2, 1/3, .4/4 }

2. several = { .5/3, .8/4, 1/5, 1/6, .8/7, .5/8 }

3. many = { .4/6, .6/7, .8/8, .9/9, 1/10 }

   PROJECT: Figure 3.2 from Schmucker

4. Note that now we are not modeling properties of an individual, but rather concepts - with the set representing a single concept, not a group of individuals.

D. It is also possible to model linguistic hedges as operations on these sets.

1. The concentrate operation (CON) operation squares the degree of membership for each element of the set. It has the effect of reducing all the degrees of membership, but affects more dramatically the members that are least in the set.

    a) Example: CON(few) = { .16/1, .64/2, 1/3, .16/4 }

    b) The linguistic hedge "very" is often modeled by CON - e.g. the set we just produced might be taken as a model of "very few".

    c) Example: If a man with height of 184 cm is a member of the set of tall men with membership degree 0.4, he is a member of the set of very tall men with membership degree 0.16.

2. The dilation (DIL) operation takes the square root of the degree of membership for each element of the set. It has the effect of increasing all the degrees of membership, but again affects more dramatically the members that are least in the set.

    a) Example: DIL(many) [to one place] =
    { .6/6, .8/7, .9./8, .9/9, 1/10 }

    b) The linguistic hedges "more or less" or "fairly" can be modeled by DIL.

3. The intensification operation (INT) increases the degree of membership of all elements with degree of membership greater than .5, and decreases the degree of membership of elements whose degree of membership is less than .5, using a function like the following:

new m(x) = 2 * (original m(x))^2 for 0 <= original m(x) < 0.5
new m(x) = 1 - 2(1-original m(x))^2 for 0.5 <= original m(x) <= 1

a) Example: INT({.3/1, ..5/2, .7/3}} = { .2/2, .5/2, .8/3 }

b) The linguistic hedge "indeed" can be modeled by INT.

4. Using these and the operations defined earlier, we can represent various other linguistic hedges as follows:

   a) "sort of" = INT(DIL(term)) ∩ DIL(INT(NOT term))

   b) "pretty" = INT(term) ∩ NOT (INT(CON(term)))

   c) "rather" = INT(CON(term)) ∩ NOT (CON(term))

   d) "slightly" = term AND NOT VERY term

   (from Schmucker)

E. It is also possible to translate a fuzzy set back to a (possibly hedged) value of a linguistic variable. This is more challenging because the fuzzy set may not correspond directly to any hedged linguistic variable value, but techniques like best fit can be used for this purpose.

IV. Uses of Fuzzy Sets

A. Fuzzy sets have been used for developing expert systems in a number of domains where the fundamental concepts are inherently fuzzy - e.g. various sorts of financial systems.

1. Before creating such a system, one would need to define appropriate fuzzy sets corresponding to the various terms to be used in the rules.

2. The rules in such a system use natural language terms like this rule (from a hypothetical expert system to help a basketball coach evaluate prospect)

   IF The player is tall
   THEN The potential is good

3. The process for using such an expert system is as follows:

   a) Fuzzification: the input data (in the form of numbers) is converted into fuzzy sets. Note that one input number may be converted into different degrees of membership in several sets - e.g. a height of 184 cm is a member of two sets (average and tall)

   b) Rule evaluation. For each rule that is applicable, the degrees of membership of the various values in the output set are multiplied by the degree of membership of the input value in the set specified in the rule antecedent.

   e.g. if the above rule were used for a person who is 184 cm high - and we used the "tall" set we've been using - then the output for this rule would consist of the set for "good" with all truth values multiplied by 0.4.

   c) Aggregation of the rule outputs. The output sets resulting from applying the rules are combined - e..g. if several rules produced output sets for "good", the sets would be combined to produce a single set for "good".

d) Defuzzification. The output set is converted back into a number using one of several possible processes.

B. Interestingly, The place where fuzzy sets have found their greatest usefulness has actually been an area that is outside the domain of AI per se: control systems.