Project Title: **Where do all those Calories come from? (Part One)**
Data Due Date: **Monday, April 3**
Report Due Date: **Wednesday, April 12**

Purpose: Explore numerical issues that arise while solving linear systems, especially systems that are composed of nearly equivalent equations.

Teams: You will work in teams of two or three, but **each student must submit their own write-up the project**. Please indicate the name(s) of your partner(s) in your write-up. Teams are limited to a maximum of three people.

Introduction

This project is designed to point out some real-world issues that arise when using linear algebra to solve practical problems, even relatively small problems. It is based on a presentation by Stephen Szydlik (University of Wisconsin Oshkosh) given at the Joint Meetings of the American Mathematical Society and the Mathematical Association of America, January 2008.

All the calories in the food we eat come from three different sources; protein, carbohydrate, and fat. Most foods sold in the United States are required to have a label that indicates the number of grams of protein, carbohydrate, and fat that are contained in each serving. Using the information from three different foods, one can set up and solve a system of equations to determine the number of calories that come from a unit of each of the these three different sources.

According the "Nutrition Facts" panel on a package of m&m's, a single serving has 10 grams of fat, 34 grams of carbohydrate, 2 grams of protein, and yields a total of 240 Calories. If we let the variables f, c, and p represent the Calories per gram of fat, carbohydrate, and protein respectively, then we obtain the equation

$$10f + 34c + 2p = 240 \tag{1}$$

In principle, if we find two more foods and set up similar equations we will have a system of three equations and three unknowns which we can solve to find values of f, c, and p. In this project you'll gather nutrition data from some different foods and use it to compute the Calories per gram from fat, carbohydrate, and protein.

Collecting Data

Each team should start by collecting nutrition information from the labels of nine different foods. Ideally you'll be able to look on a pantry shelf or head to the grocery store to do this, but if necessary you can use a website like `http://www.calorieking.com/foods/`. **Important:** Regardless of your source, only collect data from branded food (including store brands). If you use data from a website, be sure to round the data to the nearest integer, as it appears on food information labels.

Find three foods from each of the following three categories:

1. Sugary foods (e.g. chocolate bars without nuts or hard candy)
2. Energy or granola bars (e.g. power or CLIF bars)
3. Canned or frozen vegetables (e.g. canned beans, frozen squash, and stewed tomatoes)

The specific information you collect must include

- name of food,
- grams of fat per serving,
- grams of carbohydrate per serving,
- grams of protein per serving, and
- Calories per serving.

Each team must fill out the form found at the following URL before the end of the day on Monday April 3. Please remember to only *use integer values* for your numeric data.

`http://www.math-cs.gordon.edu/courses/mat232/project/projdata.html`

Investigation

Carry out the following tasks using the data you collected and report your results in a table:

1. Set up a system of three equations in three unknowns for each of the three cases and solve the systems to determine the number of calories per gram of fat, carbohydrate, and protein in each case.
2. Set up a system of three equations in three unknowns using *one* food from each of the three groups and repeat your analysis.

Referring to your tabularized data, address the following questions:

- Do your results make sense? Describe what you observe. How does the solution of each of your systems compare with the accepted solution of $[9, 4, 4]^T$?
- Pick the system that gave you the solution furthest away from the accepted solution of $[9, 4, 4]^T$ (or that had no solution at all). Change one of the smallest gram values by 1 unit (e.g. if a food has 2 grams of protein, change it to 1 or 3) and solve the system again. How much did the change impact the solution? Do you think the fact that the number of grams reported on nutrition labels are rounded to the nearest whole number of grams is significant in this experiment?

Seeking Insight

To understand what is going on in this problem, we'll look at two different systems, each with two equations and two unknowns. Consider the three equations

$$x + y = 5 \tag{2}$$
$$(1+\epsilon)x + y = 20 \tag{3}$$
$$(1+\epsilon)x - y = 20 \tag{4}$$

which we organize into two systems. The first system is composed of equations (2) and (3) while the second system uses equations (2) and (4):

System 1	System 2
$x + y = 5$	$x + y = 5$
$(1+\epsilon)x + y = 20$	$(1+\epsilon)x - y = 20$

The Greek letter ϵ (often used to represent a small quantity) gives us a "knob" that we can "turn" to make slight changes to the coefficient of x in the second equation. Notice that when $\epsilon = 0$ the first system (equations (2) and (3)) is inconsistent since the corresponding lines will be parallel.

Do the following:

1. Solve both systems to determine x and y in terms of ϵ for each of them. Pay particular attention to how ϵ appears in your solutions.
2. Graph equations (2), (3), and (4) on the same graph. Do this four times, once each with $\epsilon = -0.5, -0.2, 0.2,$ and 0.5. Use the same domain (graphing window) for all plots and make sure it is large enough to show the important features.
3. Consider how a small change in ϵ impacts the solution of system given by equations (2) and (3) and the system given by equations (2) and (4). Is your observation here consistent with the solutions you found in Step 1 above?
4. If you think of ϵ as the size of the error made when a real number is rounded to the nearest integer value, how does what you've just observed here connect to the answers you found (or didn't find) using your food data? Can you relate the geometric description of the lines in your plots to characteristics of the data collected for the various food types? If so, what can you say about this?

Assignment

Write a report that contains the following parts:

Introduction: A paragraph or two that describes what the report will cover and states the conclusion succinctly.

Investigation: This is a presentation of the work necessary to respond to the exercises and questions presented throughout this document. Write this using our textbook as a model: interleave text and equations so the text describes the mathematical work being done and is not merely sentences responding to the questions. In particular, *someone should be able to read your report without having to refer to this document*; it should be self-contained. Be sure and present your raw nutrition data, including the names of the foods you used.

Discussion: Here you should respond to the questions asked throughout this document. Rather than listing the questions and providing short answers, write a narrative that weaves in responses to the questions, and be sure an answer completely. For example, if you were asked to respond to the question "will the solution of two equations in two unknowns always exist?" in the context of talking about solutions to linear systems, you might write something like

> There are three possible outcomes when one tries to solve a system of two equations and two unknowns. The system might have a unique solution. In this case, the graph of the two equations will appear as a pair of intersecting lines. It is also possible for the system to have no solution; this happens when the lines are parallel and do not intersect. A final possibility is that there are an infinite number of solutions, in which case the lines are colinear.

The goal is to produce a document that flows well and reads easily.

Conclusion: A paragraph that summarizes your findings and restates the main result(s) from your investigation.

I will focus on the following while grading your projects:

Presentation: Your write-up for this project should be neatly handwritten or typeset. If you have difficulty figuring out how to use proper mathematical notation with your word-processor, you may leave spaces in your document and hand-write in the equations and matrices. Graphs should be presented neatly, labeled carefully and correctly. Please use a straight-edge if drawing the graphs by hand.

Completeness: I am interested in how well you can write mathematically. Be sure and include enough detail that I will not wonder how you got from one step to another.

Correctness: You should use the terminology correctly. Saying things like "the matrix has free variables" is not correct; *linear systems* can have free variables but *matrices* do not. Be sure that what you say is correct.

I suggest that you let at least one other person read your project and critique it before you create a final version to hand in. If writing is challenging for you, please consider having your paper reviewed by one of the writing tutors in the Academic Support Center.